Combinatorial Matrices' Kallista symbol:
Projective, Spectral, isotropic and multiplicative

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Abstract

Combinatorial matrices have recently been studied by Ford and the Author. We here add a fourth kind of Ford symbol, which carries both Projective and Spectral connotations. And unlike the previous three, manages to have the very simplest multiplicative property. Revealing that Combinatorial matrices' eigenvalues combine in the simplest possible way under both products and sums: an uncommon feature among square matrices. $K \times K$ Combinatorial matrices support not only K-square eigenexpansions but also 2-square eigenexpansions. This is underpinned by Combinatorial matrices mostly consisting of an isotropic block. A new style of proof for each K's Combinatorial matrices commuting with each other is also included.

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1 Introduction

Definition 1 A *Combinatorial matrix* [1] is a square matrix of size K of the following form.

$$\boldsymbol{C} := \begin{pmatrix} x + y & x & \dots & x \\ x & & & \vdots \\ \vdots & & & x \\ x & \dots & x & x + y \end{pmatrix} = y\mathbb{I} + x\mathbb{1} = (x + y(x + y, y)_K . \quad (1)$$

Remark 1 While Combinatorics often involves $x, y \in \mathbb{Z}$ or $\mathbb{N}$, Flat Geometry applications [9, 10, 11] and Linear Algebra calculations [14] extend interest to $\mathbb{Q}$ and $\mathbb{R}$. Here $\mathbb{I}$ is the identity matrix and $\mathbb{1}$ is the matrix of 1s . Also $(\ ,\ )_K$ is the Ford symbol of the zeroth kind [13, 14], a truer name for which is *irreducible symbol*, in the sense of Representation Theory. With reference to the trace–tracefree basis: now partnering $\mathbb{I}$ with $\mathbb{T}$: 1 off-diagonal and 0 on-diagonal. Whenever we work with a fixed K, we simplify this notation to $(\ ,\ )$.

Remark 2 For a fixed K, the totality of these constitute the arena $\mathfrak{C}\mathrm{M}_K(2)$: a 2-d vector space (v.s.) [13].

Remark 3 Combinatorial matrices were expanded relative to various other bases in [11, 13]. We now consider a further such, which accumulates various interesting properties.

Previously used bases all involve $\mathbb{I}$. We now bring in

$$K^{-1}\,\overline{\underline{\mathbb{1}}} = K^{-1}(1, 1) , \quad (2)$$

which we interpret as follows. As the dyad of normalized equal-entries vectors

$$\overline{\boldsymbol{n}}\,\underline{\boldsymbol{n}} . \quad (3)$$

Which is furthermore an orthogonal projector

$$\overline{\underline{\boldsymbol{P}}}_{\perp} \quad (4)$$

onto the average count acted upon's [14] 1-d *direction space*

$$\mathfrak{D}\mathrm{ir}(1) . \quad (5)$$

Whose complement is k-d *difference space* [14]

$$\mathfrak{D}\mathrm{if}(k) . \quad (6)$$

With corresponding orthogonal projector

$$\overline{\underline{\boldsymbol{P}}}_{\perp\perp} = \overline{\underline{\boldsymbol{P}}} = \overline{\underline{\mathbb{I}}} - \overline{\boldsymbol{n}}\,\underline{\boldsymbol{n}} . \quad (7)$$

This rests upon Combinatorial matrices having [14] an at least k-fold degenerate eigenvalue. Where

$$k \ := \ K \ - \ 1 \ . \tag{8}$$

Which rests in turn upon Combinatorial matrices forcing [14]. at least an $O(k)$-symmetry: a partial isotropy condition. In the generic case, this is the whole symmetry group, and the corresponding k-*d* eigenspace is *difference space*.

The only alternative to this is conflation with the remaining eigenvalue. Yielding an all-encompassing K-fold degenerate eigenspace corresponding to the $O(K)$ -symmetry: total isotropy. Now resting upon the whole matrix being $O(K)$-symmetry isotropic [14].

The above partial isotropy is of codimension 1 . On the one hand, this requires $K \ \geq \ 3$ for nontrivial realization On the other hand, this is the second-largest possible isotropy, and rapidly becomes a sizeable feature with increasing K .

Remark 4 In the N-body problem context, (5) is the *centre of mass (CoM) position label* [7, 19]. While (7) is the *CoM-removing projector* And (6) takes the form of *relative label space*

$$\mathfrak{L}\mathfrak{R}\mathrm{el}(n) \ = \ \mathbb{R}^n \ : \tag{9}$$

the space of linearly-independent (LI) separation vector labels. Labels first enter by denoting the points(-or-particles) by $1, 2, ...$. Leading to the separations themselves getting labelled by their bounding pairs of points, e.g. $1\,2$. CoM position carries both a spatial vector index and a point label index. So does the relative space of LI separations [7, 19]. But only the label part plays an active role: the full version just tensors everything with the spatial identity matrix [7, 19].

In this context, $\boldsymbol{P}$ is furthermore numerically (if not Physical-dimensionally) the *Lagrange matrix* [7, 19, 17]. The overarching theme is that upon translating to the CoM frame, all remaining degrees of freedom are relative separations. With the Lagrange matrix arising by [19] extremizing the arbitrary-translation correction to the inertia quadric with respect to its auxiliary translation variable. This is how translations, CoMs and separations are inter-related. For the general Combinatorial matrix, upon translating to the *average of the counts* that are being acted upon, all remaining freedom lies correspondingly in *differences of counts* [14].

Remark 5 The current Article's incipient idea is to take $\boldsymbol{P}_\perp$ and

$$\boldsymbol{P} \ = \ K^{-1}\,(\,k,\,-1\,) \tag{10}$$

as our basis. Which is a priori motivated by its being privileged by both of its basis matrices being projectors. The below results then build up additional a posteriori motivations.

2 Eigentheory results

Lemma 1

$$(\,x \ + \ y, \, x\,)\,\boldsymbol{P} \ = \ y\,\boldsymbol{P} \ = \ \lambda\,\boldsymbol{P} \ =: \ \lambda_k\,\boldsymbol{P} \ . \tag{11}$$

$$(\,x \ + \ y, \, x\,)\,\boldsymbol{P}_\perp \ = \ (\,K\,x \ + \ y\,)\,\boldsymbol{P}_\perp \ = \ \lambda_\perp\,\boldsymbol{P}_\perp \ =: \ \lambda_1\,\boldsymbol{P}_\perp \ . \tag{12}$$

Where λ is the k-fold degenerate eigenvalue λ_k : with algebraic multiplicity k [14]. While $\lambda_\perp$ is the complementary eigenspace's lone eigenvalue λ_1 : with algebraic multiplicity 1 .

Corollary 1

$$(\,x \ + \ y, \, x\,) \ = \ \lambda\,\boldsymbol{P} \ + \ \lambda_\perp\,\boldsymbol{P}_\perp \ . \tag{13}$$

Naming Remark 1 This prompts introducing a fourth 'Ford symbol': the *Projective symbol*

$$\{\,\lambda, \, \lambda_\perp\,\}_K \ . \tag{14}$$

Where the first entry is the $\boldsymbol{P}$ component and the second the $\boldsymbol{P}_\perp$ component. But these entries are furthermore the spectrum of eigenvalues (without multiplicities). And so another truer name for it is *Spectral symbol*. That $\leq \ 2$ distinct eigenvalues comprise the whole eigenspectrum is underpinned by Remark 3 as follows. Since Combinatorial matrices must have a $\geq \ k$-fold degenerate eigenvalue, the Pigeonhole Principle only leaves room for $\leq \ 2$ distinct eigenvalues. Since this degeneracy in turn rests upon an at least $O(k)$-symmetry – total or codimension-1 isotropy – a third name is motivated: *isotropy symbol*.

Lemma 2

$$\{ p \lambda_1, p \lambda_{2\perp} \} = p \{ \lambda_1, \lambda_{2\perp} \} . \quad (15)$$

$$\{ \lambda_1, \lambda_{1\perp} \} + \{ \lambda_2, \lambda_{2\perp} \} = \{ \lambda_1 + \lambda_2, \lambda_{1\perp} + \lambda_{2\perp} \} . \quad (16)$$

$$\{ \lambda_1, \lambda_{1\perp} \} \{ \lambda_2, \lambda_{2\perp} \} = \{ \lambda_1 \lambda_2, \lambda_{1\perp} \lambda_{2\perp} \} . \quad (17)$$

Remark 1 So passing to using our symbol preserves the simplest linearity properties, while attaining the simplest product formula. Nor is it at all usual for the eigenvalues of the product of 2 matrices to be the product of the eigenvalues of the 2 matrices. And yet by the above this holds for any 2 Combinatorial matrices of the same size... And similarly with 'sum' in place of 'product'!

Naming Remark 2 Let us celebrate by, firstly, placing a fourth name on the Combinatorial matrix symbol $\{ \, , \, \}$: *multiplicative symbol*. Secondly, by pointing out that at present this symbol 'conceptually outnumbers' our irreducible symbol $(\, , \,)$: by 4 to 1 . I.e. it is known to have multiplicative, Projective, Spectral and isotropic significance. To its rival carrying Representation-Theoretic significance. By the above confluence of properties from across Mathematics, we call $\{ \, , \, \}$ the Combinatorial matrix *Kallista symbol*. And thirdly by extending to the following.

Corollary 2

$$\sum_{i=1}^{R} \{ \lambda_i, \lambda_{i\perp} \} = \left\{ \sum_{i=1}^{R} \lambda_i, \sum_{i=1}^{R} \lambda_{i\perp} \right\} , \quad \prod_{i=1}^{R} \{ \lambda_i, \lambda_{i\perp} \} = \left\{ \prod_{i=1}^{R} \lambda_i, \prod_{i=1}^{R} \lambda_{i\perp} \right\} . \quad (18)$$

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Remark 2 Next working with the quadratic form version – acting on an input vector of numbers $\boldsymbol{K}$, we readily obtain the following results. In the N-body problem case, it turns out to be quite useful [10, 19] to take $\boldsymbol{K} = \boldsymbol{S}$: the vector of side-lengths squared. These are 1 dependency away from being LI. But there are also K of them, while we are only expecting k differences. So well-determinedness is attained.

Proposition 1 2-squares expansion.

$$||\boldsymbol{K}||_{\{ \lambda, \lambda_\perp \}}{}^2 = \lambda_\perp ||\boldsymbol{K}||_{\boldsymbol{P}_\perp}{}^2 + \lambda ||\boldsymbol{K}||_{\boldsymbol{P}}{}^2 = \lambda_\perp K^{-1} U^2 + \lambda ||\boldsymbol{K}||_{\boldsymbol{P}}{}^2 . \quad (19)$$

Proposition 2 The K-squares expansion in $\mathrm{K}(K)$ network choice of basis is as follows.

$$||\boldsymbol{K}||_{\{ \lambda, \lambda_\perp \}}{}^2 = \lambda_\perp K^{-1} U^2 + \lambda \sum_{q=1}^{k} q^{-1} Q^{-1} \left(\sum_{p=q}^{k} K_p - q K_Q \right)^2 . \quad (20)$$

Remark 3 In the N-body problem context, firstly the unit-normalized average of the counts U is realized by a quantity proportional to R : the square of the *radius of gyration*. Secondly, the first distinct K network is called the Jacobi-K [3, 17, 19] for the 4-body problem. This corresponds to [6, 12] the straight-3-path unlabelled rooted binary tree (URBT) [2]. The subsequent K networks are the corresponding straight-path URBTs for each larger path. [14] showed that these bases remain meaningful for arbitrary Combinatorial matrices. Thirdly, the above codimension-1 isotropy corresponds in the N-body setting to the $O(n)$ group of *internal rotations* alias *democracy transformations* [4].

Corollary 3

$$||\boldsymbol{\kappa}||_{\{ \lambda, \lambda_\perp \}}{}^2 = \lambda_\perp K^{-1} + \lambda ||\boldsymbol{\kappa}||_{\boldsymbol{P}}{}^2 = \lambda_\perp K^{-1} + \lambda \sum_{q=1}^{k} q^{-1} Q^{-1} \left|\left| \sum_{p=q}^{k} \boldsymbol{\kappa}_p - q \boldsymbol{\kappa}_Q \right|\right|^2 . \quad (21)$$

Remark 4 Here we have introduced the ratio variables

$$\boldsymbol{\kappa} := U^{-1} \boldsymbol{K} . \quad (22)$$

Remark 5 On the one hand, the K-squares expansion uses the whole eigenspectrum: including algebraic multiplicity. And rests upon the following decomposition of the v.s. $\mathfrak{V}(K)$ of vectors acted on by our size-K Combinatorial matrices.

$$\mathfrak{V}(K) \; = \; Im_{\boldsymbol{P}_\perp}(\mathfrak{V}(K)) \; \oplus \; Im_{\boldsymbol{P}}(\mathfrak{V}(K)) \;\; = \;\; \mathfrak{D}\text{ir}(1) \; \oplus \; \mathfrak{D}\text{if}(k) \; . \tag{23}$$

On the other hand, the 2-squares expansion uses the following 2-d v.s. decomposition.

$$\mathfrak{C}\text{M}_K(2) \; = \; \mathfrak{D}\text{ir}(1) \; \oplus \; \mathfrak{D}\text{if}(1) \; . \tag{24}$$

3 Examples

Examples 1-7: Using our Kallista symbol, The CoM-removing projector itself is

$$\boldsymbol{P} \; = \; \{\, 1, 0 \,\} \; . \tag{25}$$

Its complement is

$$\boldsymbol{P}_\perp \; = \; \{\, 0, 1 \,\} \; . \tag{26}$$

The zero matrix is

$$\mathbb{O} \; = \; \{\, 0, 0 \,\} \; . \tag{27}$$

And the identity matrix is

$$\mathbb{I} \; = \; \{\, 1, 1 \,\} \; . \tag{28}$$

The previous two are both examples of isotropic matrices, the general case of which is

$$r \{\, 1, 1 \,\} \; , \quad r \; \in \; \mathbb{R} \; . \tag{29}$$

The fundamental 2-simplex matrix, shared by the triangle inequality, the cosine rule, and Heron's formula, [8, 9, 10, 11] is

$$\boldsymbol{F} \; = \; \{\, \text{-}2, 1 \,\} \; . \tag{30}$$

The Apollonius involutor [8, 10, 11] is

$$\boldsymbol{J} \; = \; \{\, \text{-}1, 1 \,\} \; . \tag{31}$$

Example 8 Of sums of K squares. In the $K \; = \; N \; = \; 3$-body problem context, Proposition 2 returns $Aniso^2 \; + \; Anelp^2$ for its last 2 squares [8, 9, 10, 19]. Standing for *anisoscelesness*: departure from isoscelesness. And departure from being in equilateral proportion, with reference to the base to median ratio. In terms of side-lengths^2 A, B, C ,

$$Aniso \; = \; \frac{A \; - \; B}{\sqrt{2}} \; , \quad Anelp \; = \; \frac{A \; + \; B \; - \; 2\,C}{\sqrt{6}} \; . \tag{32}$$

Generalizing from sides-lengths^2 in the 3-body problem to differences of primary objects in Combinatorics, we write U in place of R . Not $Aniso$ but Ind , in the sense of induced from a $K \; = \; 2$ difference. And not $Anelp$ but $Comp$, in the sense of orthonormal complement.

Proposition 2 recovers as a subcase

$$||\boldsymbol{K}||_{(\,x \; + \; y, \, x\,)}{}^2 \;\; = \;\; \frac{3\,x \; + \; y}{3} \, U^2 \; + \; \frac{y}{2} \left[(\, A \; - \; B\,)^2 \; + \; \frac{(\, A \; + \; B \; - \; 2\,C\,)^2}{3} \right] \; . \tag{33}$$

Which can also be written in the following generalized Euler 3-cycle form.

$$\begin{aligned} ||\boldsymbol{K}||_{(\,x \; + \; y, \, x\,)}{}^2 \;\; &= \;\; \frac{3\,x \; + \; y}{3} \sum_{\text{3-cycles}} A\,(\, A \; + \; 2\,B\,) \; + \; \frac{2\,y}{3} \sum_{\text{3-cycles}} A\,(\, A \; - \; B\,) \\ &= \; \sum_{\text{3-cycles}} A\,[\,(\, x \; + \; y\,)\, A \; + \; 2\,x\,B\,] \; . \end{aligned} \tag{34}$$

Setting x and y to match $\boldsymbol{F}, \boldsymbol{J}$ and $\boldsymbol{P}$ in turn recovers some previous results from [8, 11]. In particular, since $\boldsymbol{P}$ is itself the projector, the eigenvalue $\lambda_\perp$ corresponding to $\boldsymbol{P}_\perp$ is 0 . And so the $U \; = \; R$ contribution to $\boldsymbol{P}_\perp$ drops out entirely.

4 Algebras

Combinatorial matrix projectors' commuting monoid $\boldsymbol{P}$

$\bullet$	$\mathbb{I}$	$\boldsymbol{P}$	$\boldsymbol{P}_\perp$	$\mathbb{O}$
$\mathbb{I}$	$\mathbb{I}$	$\boldsymbol{P}$	$\boldsymbol{P}_\perp$	$\mathbb{O}$
$\boldsymbol{P}$	$\boldsymbol{P}$	$\boldsymbol{P}$	$\mathbb{O}$	$\mathbb{O}$
$\boldsymbol{P}_\perp$	$\boldsymbol{P}_\perp$	$\mathbb{O}$	$\boldsymbol{P}_\perp$	$\mathbb{O}$
$\mathbb{O}$	$\mathbb{O}$	$\mathbb{O}$	$\mathbb{O}$	$\mathbb{O}$

Figure 1:

Proposition 3 Under multiplication, $\boldsymbol{P}$ and $\boldsymbol{P}_\perp$ alongside the identity $\mathbb{I}$ and zero $\mathbb{O}$ form the commutative monoid [5] whose times table is in Fig 1.

Proposition 4 The above four inputs also form a zero-commutator algebra.

Remark 1 Proposition 3's times table follows from just the identity property, the projector property and projector complementarity. Proposition 3's inputs reflect that, on the one hand, we can freely append an identity element. On the other hand, we are forced to include the zero since we discover it as the product of our 2 projectors.

Remark 2 For Proposition 4, the only nontrivial bracket to check is

$$[\,\underline{\overline{\boldsymbol{P}}}\,,\,\underline{\overline{\boldsymbol{P}}}_\perp\,] \;=\; \underline{\overline{\boldsymbol{P}}}\cdot\underline{\overline{\boldsymbol{P}}}_\perp \;-\; \underline{\overline{\boldsymbol{P}}}_\perp\cdot\underline{\overline{\boldsymbol{P}}} \;=\; \underline{\overline{\mathbb{O}}} - \underline{\overline{\mathbb{O}}} = \underline{\overline{\mathbb{O}}}\,.$$

Where step 1 is by the definition of commutator. And step 2 makes 2 uses of complementarity.

Remark 3 The current Section is the generalized Combinatorial matrices' counterpart of the triangle (or more generally 2-simplex) algebras presented in [11]. And of various quadrilateral algebras presented in [15, 16, 18]. A large Algebraic simplification in [11] turns out to be underpinned by the $K = N = 3$ subcase of $\boldsymbol{P}_\perp$.

5 Conclusion

Remark 1 A first motivation for our Kallista symbol for Combinatorial matrices is that it corresponds to the Projective choice of basis.

Remark 2 Our second motivation is Spectral: our basis displays the eigenvalues as its components. And clarifies that these combine particularly simply for Combinatorial matrices under both addition and multiplication. General square matrices' eigenvalues do not have these properties! By which our basis displays a 2-term eigenexpansion. This is rendered possible by all Combinatorial matrices having an eigenvalue that is at least k-fold degenerate by symmetry. Due to the underlying at-least $O(k)$-symmetry, and thus a total or codimension-1 isotropy, giving a third Group-Theoretic motivation.

Remark 3 Using our new basis, the product rule (17) and the commutative monoid concurrently manage to take particularly simple forms. In contrast to the various basis choices in [11], the current Article's commutative monoid's is distinctive by its times table necessarily involving the zero. That compatible Combinatorial matrices always commute was recently pointed out by Ford [13]. Proposition 4 however provides an alternative style of proof for this! I.e. every Combinatorial matrix can be written as the weighted sum of a complementary pair of projectors. Which commute with each other by complementarity, and so all compatible-sized Combinatorial matrices commute with each other. This paragraph lays out our fourth multiplicative motivation for the Kallista symbol.

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