BUNDLES. 2

(is Diff geom 8)

6

§1 GENERALIZED BUNDLES

Bundles [Husemoller]: → to become 10 sides of notes: his Ch 2.

These generalize fibre bundles by dropping the local product structure.

There are no longer fibres.

(E, B, F, G, π) to (E, π, B) $\qquad \pi : E \to B$

fibre bundle

The role of F which does survive is that of preimage of π: $\pi^{-1}(x)$.

$\pi^{-1}(x)$ are all the same in the fibre bundle case; corresponding to all F's being the same.

In the generalized bundle setting, the $\pi^{-1}(x)$ are no longer all the same.

['The same' means, in particular, homeomorphic, though it can mean more, e.g. isomorphic for a vector bundle.

One can still call $\pi^{-1}(x)$ a fibre, under the understanding that these need no longer all be equal.

The notion of bundle morphism carries over straightforwardly

$$\begin{array}{ccc} E & \xrightarrow{u} & E' \\ \pi\downarrow & & \downarrow\pi' \\ B & \xrightarrow{f} & B \end{array}$$

[Husemoller]

So does the notion of subbundle: (E', π', B') is a sub-bundle of (E, π, B) if $B' \subseteq B$, $E' \subseteq E$ and $p' = p|_{E'}$.

So does the notion of cross-section: $s : E \to B$ [opposite way round to $\pi : B \to E$]

s.t. $\pi(s(b)) = b$ for each $b \in B$. Equivalently, $s(b) \in \pi^{-1}(b)$.

Examples (of bundles other than fibre bundles).

1) Fibred manifolds: surjective submersions $\pi : E \to B$

differentiable mapping with surjective tgt mapping $T_y\pi : T_y E \to T_{\pi(y)} B$

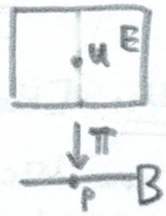
2) Fibrations: have the fibres $\pi^{-1}(b_1)$ homot $\simeq$ $\pi^{-1}(b_2)$.

3)

Further generalization: bundle objects in any category (see next folder)

use this as an essay plan
to remodel the other side's contents ...

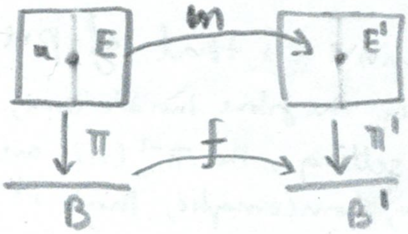
The more logically ordered pedagogy : (used in Top folder!)



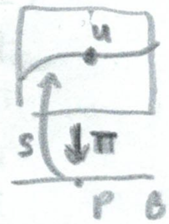
First study $\pi : E \longrightarrow B$.

~ spaces which project down onto lower-d spaces which can also be viewed in reverse as 'higher-d bundle total spaces', built over the lower-d base spaces

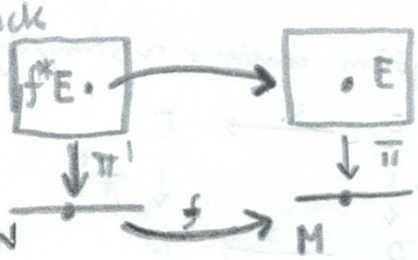
These have morphisms



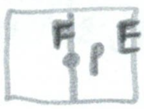
And some notion of section ~ a map in opp dir to π but with 1 to 1 nes, whereas π loses info.



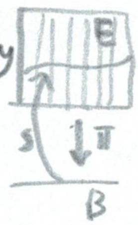
And a notion of induced bundle/ pull back



Then further introduce a local product structure, giving the fibre concept.

This is depicted by 

, and then fibre bundle sections cut each fibre ! by



Then one case of this is $F = G$, for which G acts on itself. This gives the principal bundle notion.

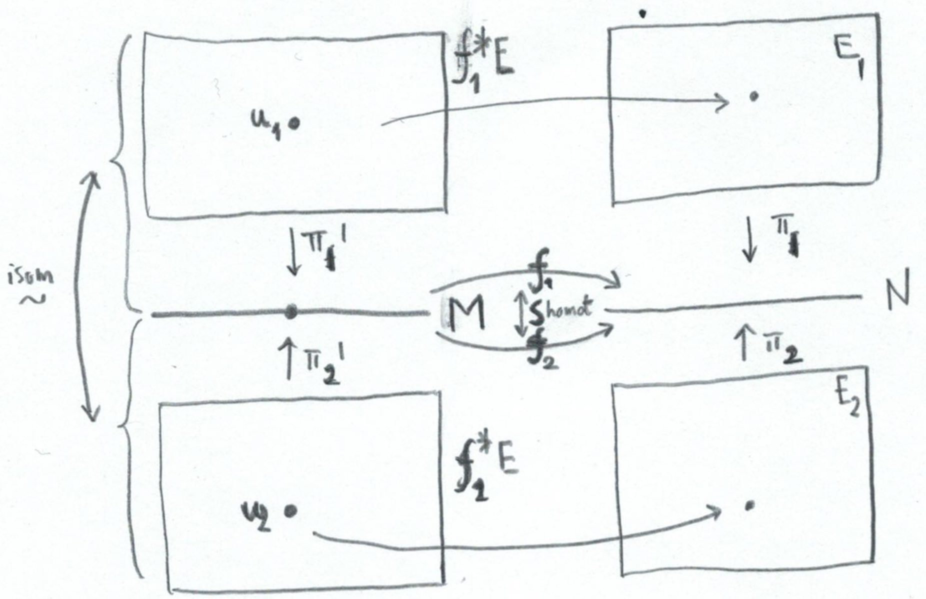
Moreover, one can introduce G without F by having B be E/G : an orbit space (see §4)
This is only sometimes a fibre bundle: if G acts freely.
If not, one has non-homeomorphic objects involved fibres...

Finally, one can imagine F and G distinct, but with G still acting on the F which has taken over one of G's two roles. This gives the associated bundle role, which is the general fibre bundle notion.

These last two are both then very useful in modelling gauge theory, justifying their introduction as additional layers of structure.

[* Restricting attention to $B = B'$ case is also very significant subcase : see top notes.]

Universal bundles rather begs a picture:



This isom ~ $\Leftrightarrow$ this homot ~.