

63.0 From Topological groups to Lie algebras

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Topological group: group & topological space.

Lie group: group & differentiable manifold

group generators are represented by derivative operators.
These involve looking locally around the identity

Lie algebra

~ still makes use of differentiable structure,
but no longer of global topology.

Uses: is simpler than Lie group.

Is in tight correspondence with Lie group.

Will already do to get Rep Theory

Are linear spaces,

but with a 3rd composition operation $[,]$ in addition to $+$ and $\cdot$. Hence 'Algebra'

[Linear spaces are fine for most aspects of Rep Theory]

We then return to the global subtleties of Lie algebras
and precisely which Lie groups correspond to each Lie algebra.

It is however possible to consider Topological groups locally, without making reference to differentiable structure...

In the absence of a + specific name, I term this approach 'Topological Lie algebra'.

It still has $[,]$, defined for matrixes rather than for differential operators.

Hence we have

Topological space structure

Topological manifold structure

Differentiable manifold structure

Topological group

Lie group

Topological Lie algebra
on local chart
around origin

Lie algebra

Tgt space

→ Q: how much does a topological Lie algebra need to be Hausdorff and 2nd countable?